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## Special issue on global flow instability and control

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**Abstract** This special issue is the second on the topic of “Global Flow Instability and Control,” following the first in 2011. As with the previous special issue, the participants of the last two symposia on Global Flow Instability and Control, held in Crete, Greece, were invited to submit publications. These papers were peer reviewed according to the standards of the journal, and this issue represents a snapshot of the progress since 2011. In this preface, a sampling of important developments in the field since the first issue is discussed. A synopsis of the papers in this issue is given in that context.

**Keywords** Global linear instability · Flow control · Transition · Turbulence

### 1 Progress since the first special issue

A special issue was included in Volume 25 of this journal in 2011, devoted to the first four symposia on Global Flow Instability and Control, held in Crete, Greece, since 2001. A number of important developments in the field, first presented as oral contributions in the meetings, were discussed in eighteen peer-reviewed articles contained in that volume. In the time elapsed, these articles have generated over 600 citations in Google Scholar, an indication of the rapid pace by which the field is expanding. For this reason, after the Crete VI event in 2015, it was decided that a second special issue on the topic was warranted; this volume is the result. The aims of the issue are to document both fundamental progress and the gradual maturation of more established techniques used to analyse and control instabilities of flows in complex geometries.

Amongst the evident trends, it is fair to say that the continuing exponential growth of computing power has expanded the range of feasible problems, which has resulted in calculations that are increasingly relevant to applications. Other important developments include: short-time horizon linear analysis of laminar flows, as

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well as asymptotic analysis of turbulent flows with multiple inhomogeneous spatial directions; experimental demonstration of control of turbulent flows; further development of theoretical tools for flow control; as well as new numerical methods that permit efficient recovery of accurate base flows to be analyzed. We see both matrix-free and full-matrix techniques co-existing, and the maturation of adjoint techniques, both for optimization and for transient growth calculations. These have followed the path of stability problems, being generalized to complex geometries.

Global transient growth analysis, in the sense of the calculation of transient growth in a complex geometry, emerged from the third Crete symposium in 2005 [27] and first appeared in the literature in a conference contribution [28], followed by work in the same vein in canonical and more applied geometries [1, 5, 19, 26]. The mechanism presented in the transient growth analysis of flow around a circular cylinder by Abdessemed et al. [1] was recently found by He et al. [12] to be relevant to flow over stalled aerofoils. Such transient growth calculations may be viewed as a special case of more general linear, and nonlinear, adjoint-based optimization and control problems presently studied in fluid mechanics. Interest in the adjoint field has a long history [15], and this type of approach has become a workhorse in the stable of methods available to the computational fluid dynamicist.

One of the most significant, but subtle, changes in perspective has been regarding stability calculations about a time-averaged flow. The debate around the meaning of the time- or space-averaged turbulent flow in stability calculations dates back to Malkus [17]. This question was revisited in 2006 [3, 23] where it was shown that a stability calculation around the temporal mean flow is able to correctly predict the shedding frequency of flow past a cylinder away from the Hopf bifurcation point (whereas the same calculation about the continued fixed point does not). While a rigorous justification for this procedure was lacking, its success in some cases has led to the widespread use of the mean flow for such calculations [30]. Possible resolutions to this conundrum came in a few different guises. A workable approach to resolving this situation for the cylinder flow, involving the limited reintroduction of nonlinearity, was given by Mantić-Lugo et al. [18]. In that work, the model is composed of a set of coupled equations governing the mean flow together with its most unstable eigenvector with finite size; the amplitude is determined by requiring the eigenvalues of the Jacobian calculated around the mean flow to have maximum real part of zero. It remains unknown for which flows this ‘RZIF’ property [34] is the appropriate condition.

The use of the mean in similar calculations arises naturally in what has become known as resolvent analysis; the relevant mathematical formulation appeared first in the context of turbulent flows in a work by McKeon and Sharma [21] and has recently been explicitly related back to this topic by Gómez et al. and Beneddine et al. [4, 11]. Resolvent analysis has followed the path of stability and transient growth calculations towards complex geometries; what might well be called global resolvent modes are presented in the same sources. The Koopman mode view of such mean flow stability calculations was given in [22]; it turns out the Koopman mode and dynamic mode decomposition formulations are closely related to resolvent analysis [29, 33]. Even within the school of resolvent analysis, there are various interpretations. The first is to model the nonlinearity as a stochastic forcing to the linear system. Into this set, we may place [35] and early works of the same group. The second ([21] for example) views the system deterministically, seeking a self-consistent solution of both the linear and nonlinear fluctuation equations. The so-called resolvent modes arise from this perspective as an optimal projection in which a single representative trajectory may be expanded. A recent theoretical innovation by Towne et al. [33] extends this analysis to ensembles which may prove key to reconciling the statistical and deterministic formulations, at least for statistically stationary flows.

The shift in perspective from behaviour around a fixed point to mean flow analysis can cause conceptual difficulties for the straightforward application of feedback control techniques straight from the control engineer’s library. Such techniques usually assume sufficient actuation to maintain steady desired operating point, which is typically false for a turbulent flow. Nonetheless, traditional feedback control techniques have often proved effective, as the study of Dalla Longa et al. [6] in this issue shows. To avoid the open theoretical questions of fluid mechanics, we may use data-driven approaches (machine learning in the broadest sense). This class of approach is represented in this issue [14] and will no doubt eventually benefit from the vast industrial effort being put into solving big data problems. How our new understanding and approaches will fuse to allow new capability in flow control remains over the horizon.

## 2 Contributions to the present volume

A subset of the body of talks presented and ideas discussed during the last two Crete meetings has been submitted as peer-reviewed contributions to this special issue. On the theme of stability, TriGlobal stability

analyses of flows past a freely rotating sphere [9], the wake of a three-dimensional wing [13] and boundary layers perturbed by streamwise vortices [20] are discussed. A unified perspective of instability in rotating Bödewadt, Ekman and von Kármán layers has been presented [7]. Instability of variable density flows in three-dimensional containers has been analysed numerically [10,25] while a combined analytical and experimental effort discussed instability of compressible spanwise homogeneous cavity flow [31]. In the flow control theme, a linear feedback control approach was demonstrated to reduce the pressure drag of a D-shaped bluff body [6] and cluster-based control of a separating flow over a smoothly contoured ramp has been discussed [14]. The differentially heated cavity was used as a prototype application to demonstrate adjoint optimization of natural convection problems [25]. Finally, contributions to numerical aspects of flow instability and control analysis presented in this volume include a criterion for optimal grid adaptation associated with error sensitivity to grid refinement [16], as well as a novel framework for the computation of unstable steady basic flows [8], alternative to the well-established selective frequency damping method [2]. A boundary condition for the collocated solution of the two- and three-dimensional eigenvalue problems on collocated grids was presented [32] and a framework for the stability analysis of flows obtained by Lattice Boltzmann methods [24] was also discussed.

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